

## Frames of Reference

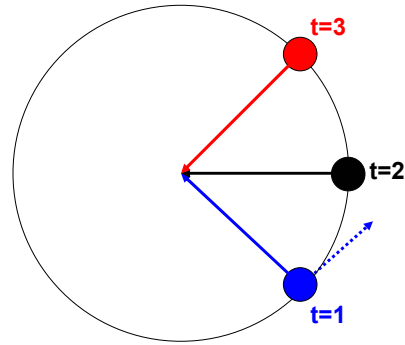
- Newton's laws of motion are valid in a coordinate system that is fixed in space.
- A coordinate system fixed in space is known as an **inertial** (or **absolute**) **frame of reference**.
- A coordinate system that is not fixed in space, such as one defined with respect to the rotating earth, is a **noninertial frame of reference**.
- Because we are interested in atmospheric and oceanic motions from an earth-based perspective, we must formulate the laws of motion in the noninertial frame of reference defined with respect to the rotating earth.

## Apparent Forces

- In an inertial reference frame, a body at rest or in uniform motion has no net forces acting on it.
- A body at rest or in uniform motion relative to the rotating earth is not at rest or in uniform motion relative to a coordinate system fixed in space.
- To reconcile Newton's laws with the noninertial reference frame of the rotating earth, two **apparent forces** must be introduced.
- These are the **centrifugal force** and the **Coriolis force**.

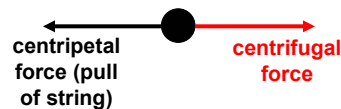
## Centrifugal Force

- Consider an object that is whirled in a circular motion at the end of string.
- In an inertial frame of reference, this object is being accelerated toward the center of the circle.
- This acceleration is called **centripetal acceleration**, and the force that causes it is the pull of the string.



## Centrifugal Force

- Consider a frame of reference that is moving with the object at the end of the string.
- In this noninertial frame of reference, the object appears stationary despite the pull of the string toward the center of the circle.
- An additional apparent force must be included to exactly balance the force of the string.
- This apparent force is the **centrifugal force**.



no net force  
↓  
no acceleration  
(in rotating frame  
of reference)

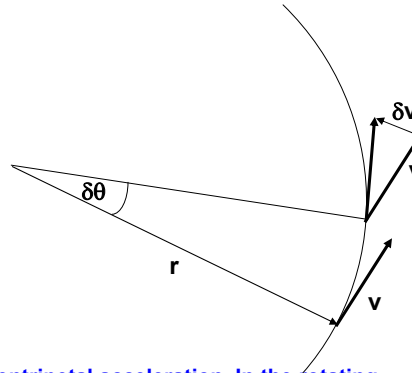
As in the previous example, an object is attached to a string and whirled in a circle of radius  $r$  at a constant angular velocity  $\omega$ . In a very small time increment  $\delta t$  the object rotates through an angle  $\delta\theta$ .

$$\frac{d\vec{V}}{dt} = |\vec{V}| \frac{d\theta}{dt} \left( -\frac{\vec{r}}{r} \right)$$

$$|\vec{V}| = \omega r \quad \frac{d\theta}{dt} = \omega$$

$$\frac{d\vec{V}}{dt} = -\omega^2 \vec{r}$$

This is the centripetal acceleration. In the rotating frame of reference it is balanced by the centrifugal acceleration, which has equal magnitude but opposite sign.



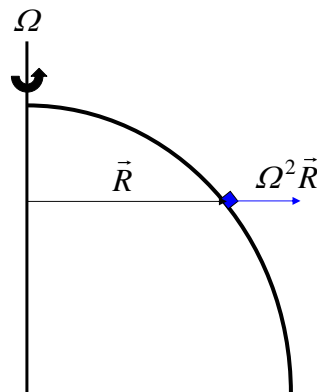
## Centrifugal Acceleration on Earth

In a rotating frame of reference, an object at rest on the earth's surface is subject to a centrifugal acceleration

$$\Omega^2 \vec{R}$$

$\Omega$  is the angular speed of rotation of the earth ( $= 7.292 \times 10^{-5} \text{ rad s}^{-1}$ ).

$\vec{R}$  is the position vector from the axis of rotation to the object.

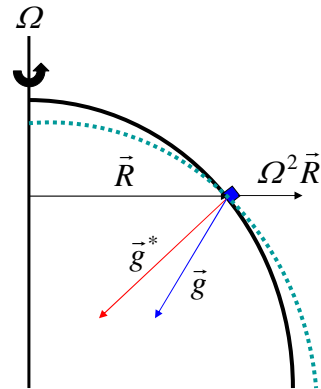


## Gravity Force

For an object at rest on the earth's surface, the centrifugal force partly balances the gravitational force, thus we define a gravity force (a.k.a. gravity)  $\vec{g}$

$$\vec{g} \equiv \vec{g}^* + \Omega^2 \vec{R}$$

Gravitational force is always directed toward center of earth; gravity is not, except at poles and equator.

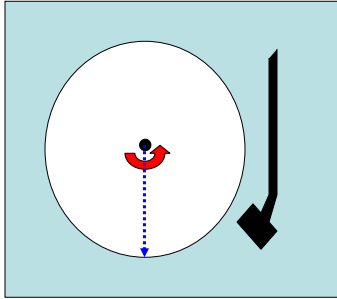


The earth has adjusted to this balance of forces by taking the shape of a spheroid with an equatorial bulge.

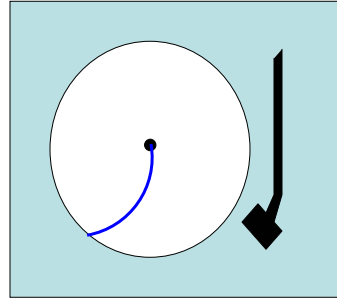
## Coriolis Force

- We can apply Newton's second law of motion in a rotating coordinate system to an object at rest with respect to that rotating reference frame by including the centrifugal force, which is an apparent force.
- If the object is moving with respect to the rotating reference frame, an additional apparent force is required for Newton's second law to be valid. This is the **Coriolis force**.

On a rotating turntable, draw a line that is straight when viewed from an inertial frame of reference.



In the rotating reference frame, the resulting line curves to the right, as can be seen when the turntable is stopped.



An object moving in a straight path in an inertial frame of reference will follow a curved path with respect to a rotating frame of reference. The apparent force that deflects the object is the **Coriolis force**.

## Coriolis Force

Consider an object on the earth's surface that is **moving in an eastward direction** (same direction that earth rotates) at a speed  $u$ . The object is now rotating faster than the earth, so the centrifugal force acting on it increases to

$$\left(\Omega + \frac{u}{R}\right)^2 \vec{R} = \Omega^2 \vec{R} + \frac{2\Omega u \vec{R}}{R} + \frac{u^2 \vec{R}}{R^2}$$

total centrifugal force
centrifugal force due to earth's rotation
deflecting forces

For large-scale atmospheric motions, this term is much smaller than the term to its left and can be neglected.

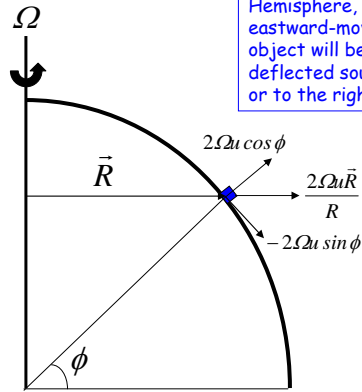
The deflecting forces act outward in a direction perpendicular to the axis of rotation.

## Coriolis Force

The Coriolis force that results from zonal (i.e., east-west) motion can be divided into components in the vertical and meridional directions.

$$\left(\frac{dv}{dt}\right)_{Co} = -2\Omega u \sin \phi$$

$$\left(\frac{dw}{dt}\right)_{Co} = 2\Omega u \cos \phi$$



In the Northern Hemisphere, an eastward-moving object will be deflected southward, or to the right.

This derivation and result applies only for zonal motion. (See Holton section 1.5 for meridional motion.) A more general way to represent apparent forces is required.